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DESIGN OF TURBOFAN ENGINE CONTROLS USING OUTPUT FEEDBACK REGULATOR THEORY

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Abstract

A multivariable control design procedure based on output feedback regulator (OFR) theory is applied to the F100 turbofan engine. Results for the OFR design are compared to a design based on linear quadratic regulator (LQR) theory. This LQR design was obtained as part of the F100 Multivariable Control Synthesis (MVCS) program. In the MVCS program the LQR feedback control was designed in a reduced dimension state space and then applied to the original system. However, the OFR feedback control is designed in the full order state space and thus eliminates any need for model reduction techniques. Using the performance measure and control structure of the MVCS program LQR design, an equivalent OFR feedback control is obtained. The flexibility of the OFR as a control design procedure is demonstrated and differing feedback control structures are evaluated.

INTRODUCTION

Increased performance requirements, more complex engine configurations, and the possibility of onboard digital engine controllers have generated interest in advanced, multivariable, engine control systems. To design these advanced control systems engine manufacturers, as well as NASA and the Air Force, are investigating multivariable design techniques (refs. 1 to 5). One program, the F100 Multivariable Control Synthesis (MVCS) program, was initiated to study the applicability of linear, quadratic regulator (LQR) theory to the design of F100 turbofan engine controls (ref. 6). One aspect of the program included obtaining constant feedback gains for regulation of engine steady-state conditions.

Engine data in the form of a nonlinear digital simulation were used to generate linear models, each valid for small perturbations about an operation point. However, the states corresponding to the linear models are not all physically measurable for use in the feedback control law as would be required by LQR theory. Estimation of the unmeasurable states from a measurable subset was judged, within the program hardware constraints, to be too complex to implement. Thus, the control design procedure selected first reduced via dominant mode approximation techniques (ref. 6) the order of the model state space to include only a measurable subspace. Then a state feedback regulator was designed using LQR theory in the reduced state space. This procedure gave an implementable control design. Moreover, a real-time hybrid computer simulation evaluation (ref. 7) has demonstrated (1) the ability of the resultant control to perform the prescribed control function and (2) the flexibility of the LQR design process for determining engine controls.

However, the model reduction process, or more specifically the selection of the measurable subspace, was not straightforward and complicated the overall design procedure. In particular the selection of the reduced state space defines a trade-off between control performance and the control implementation complexity. A definitive answer to the adequacy of this trade-off would require many iterations through the design procedure and, therefore, many model reductions. Additionally, there is no a priori assurance that any approximation technique selected will yield good (or even acceptable) results when the resultant control design is applied to the original system. Therefore, it is desirable to determine a design process that would incorporate the utility of the LQR theory but would eliminate the model reduction step.

The purpose of this paper is to propose such a design process based on output feedback regulator (OFR) theory (ref. 9) and to investigate the acceptability of this process for designing engine controls. The OFR design process incorporates a linear, quadratic formulation but would use the full order linear model directly to determine constant feedback gains for an appropriate output vector (some vector in the measurable subspace). Thus, the model reduction step is eliminated. Additionally, the OFR formulation allows an efficient study of the performance trade-off important in any realistic design. These benefits are not obtained without some cost, however. Solutions to the OFR problem satisfy only a necessary optimality condition and therefore are suboptimal when compared to the necessary and sufficient conditions of the full state feedback LQR formulation. Recall, however, that the LQR process as applied in the MVCS program used a reduced order state space and, therefore, is also suboptimal with respect to the original system. Thus, OFR suboptimality is not regarded as too significant a problem. More importantly, OFR solutions are not guaranteed to exist for any arbitrary output vector. Thus, OFR solutions must be shown to exist for the desired control structure before proceeding with the design.

The OFR design process was investigated by applying it to a 17th order linear, operating point

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model of the F100 turbofan engine. This model represents an engine intermediate power condition in a sea level, static environment. First, a brief review of OFR theory is given. Then, comparisons are made between the resultant OFR feedback control and the MVCS program LQR control designed at the same condition. Finally, the flexibility of the OFR approach is demonstrated by evaluating the improvement in performance when additional engine variables are measured and included in the control configuration.

MATHEMATICAL DEVELOPMENT

Given a time invariant linear system

$$\dot{x} = Ax + Bu, \qquad x(0) = x_0 \tag{1}$$
$$y = Cx$$

where the initial state, x_0, is a zero-mean random variable with covariance, X_0. The OFR problem is to find the time invariant feedback law

$$u = -Fy \tag{2}$$

which minimizes

$$J = E\left\{\frac{1}{2}\int_0^{\infty} (x^TQx + u^TRu)dt\right\} \tag{3}$$

with

$$Q \geq 0, \qquad R > 0 \tag{4}$$

Necessary conditions for optimality and computational solutions are derived for this problem statement in references 8 and 9 and for its discrete time counterpart in reference 4. The necessary conditions are

$$KA_0 + A_0^TK + Q + C^TF^TRFC = 0 \tag{5a}$$

$$LA_0^T + A_0L + X_0 = 0 \tag{5b}$$

$$F = R^{-1}B^TKLC^T(CLC^T)^{-1} \tag{5c}$$

where

$$A_0 = A - BFC \tag{6}$$

The solution of (5) gives F, and the suboptimal value of J for this F is given by

$$J = \frac{1}{2}\,\mathrm{Tr}(KX_0) \tag{7}$$

OFR DESIGN FOR F100

The OFR design procedure outlined in the preceding section was applied to the 17th order linear time invariant model of engine operation point, small perturbation, dynamics for an intermediate power condition in a sea level, static environment. The input and full state vectors that represent the F100 engine variables are given in Table I. The numerical description of this model is given in reference 6. For comparison purposes the same performance weightings used for the MVCS program LQR design at this condition were used in the OFR design. These numerical weightings are also given in reference 6.

The LQR and OFR procedures used the same input vector but the LQR approach selected a reduced state vector as shown by equation (8).

$$\hat{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \\ x_{17} \end{bmatrix} \tag{8}$$

If we let the output vector y be the reduced order state vector

$$y = \hat{x} = Cx \tag{9}$$

then the regulator structure when applied to the original system is the same for both the OFR and LQR designs. Computationally, the feedback matrix, F, of equation (2), was solved by approximating the continuous system of (1) by its discrete counterpart and applying the numerical algorithms of reference 4. The sampling period was selected as $T = 0.0001$ secs/cycle to give a close approximation to the continuous system since the open loop eigenvalues are greater than -600 rad/sec. With this approximation the output feedback matrix could be directly compared to that obtained via the LQR design. Also, the initial condition covariance matrix was somewhat arbitrarily set as $X_0 = I$ for all the OFR designs of this paper.

The numerical procedure is iterative in nature and requires a stabilizing output feedback matrix as an initial guess. Since the F100 linear models are open loop stable, an initial guess of zero for each feedback element can be conveniently selected to start the iterative process. With this initial guess the output feedback matrix, F, was found for this model quite easily. The feedback matrices and the closed loop eigenvalues for the OFR and LQR approaches are compared in Table II. Also, included are the final cost function values. The results compare quite favorably and it can be seen that control performance equivalent to the LQR design could have been directly determined by the OFR design process.

OFR DESIGN FLEXIBILITY

The OFR design procedure offers the usual features of LQR design, plus the additional capability of investigating different structures (via the C matrix) without altering the basic dynamic model of the system (A and B). To demonstrate this flexibility the F100 model was studied to determine if different combinations of measurable engine

variables could result in better control performance or a simplified control structure, given the previously defined performance weightings. With the performance measure of (7) it is possible to compare control effectiveness for different feedback control structures.

Three different structures were compared for the given engine model. The output vectors for these structures are

Structure #1

$$y = \hat{x} \tag{10}$$

Structure #2

$$y = \begin{bmatrix} x_1 \\ x_3 \\ x_5 \\ x_{17} \end{bmatrix} \tag{11}$$

Structure #3

$$y = \begin{bmatrix} \hat{x} \\ \cdots \\ x_{14} \end{bmatrix} \tag{12}$$

Structure 1 represents the LQR design actually implemented for F100 control and serves as a reference point. Structure 2 investigates the importance of the compressor speed measurement on control effectiveness by deleting it from the feedback structure. For example, insight can be gained here with respect to the loss of the compressor speed sensor given the implementation of Structure 1. The changes in engine performance and feedback matrix elements between Structures 1 and 2 will indicate the relative importance of the compressor speed measurement for feedback control. Structure 3 investigates the importance of fan turbine exit temperature as a feedback element in the control structure when compared to the control of Structure 1.

Results for these three output structures are given in Table III. There is a significant variation in the value of the cost (J) between control Structures 1 and 2. Also there are differences in the appropriate feedback matrix elements. It can be concluded that compressor speed is a significant variable for regulation purposes for the F100 model given the model and performance weightings of the previous section.

A comparison of results for control Structures 1 and 3 indicate that the inclusion of turbine exit temperature in the feedback control does not significantly increase the value of the cost function (J). Thus, the difficulty in making this additional measurement could not be justified. This study is by no means exhaustive, but rather an indication of the facility with which the complexity-performance trade-off can be handled using the OFR design procedure.

SUMMARY

This paper has presented a multivariable control design procedure based on OFR theory that can be used to design operating point controls for jet engines. The procedure utilizes the benefits of a linear, quadratic approach but eliminates the model reduction step required by LQR theory when the number of measurable engine variables is less than the dimension of the linear model state space. Additionally, sensor failure and complexity-performance trade-off studies, in the form of differing feedback structures, can be handled quite readily with this formulation. Results were obtained by applying the OFR procedure to a linear 17th order model of the F100 turbofan engine and compared to those obtained by a reduced dimension LQR approach.

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TABLE I. DEFINITIONS OF CONTROL AND STATE VECTORS FOR F100 LINEAR MODEL

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} \text{Commanded fuel flow} \\ \text{Nozzle jet area} \\ \text{Inlet guide vane position} \\ \text{High compressor variable stator position} \\ \text{Compressor bleed flow} \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \end{bmatrix} = \begin{bmatrix} \text{Fan speed} \\ \text{Compressor speed} \\ \text{Compressor discharge pressure} \\ \text{Inter-turbine volume pressure} \\ \text{Augmentor pressure} \\ \text{Fan inner diameter discharge temperature} \\ \text{Duct temperature} \\ \text{Compressor discharge temperature} \\ \text{Fast response burner exit temperature} \\ \text{Slow response burner exit temperature} \\ \text{Burner exit total temperature} \\ \text{Fast response fan turbine inlet temperature} \\ \text{Slow response fan turbine inlet temperature} \\ \text{Fan turbine exit temperature} \\ \text{Duct exit temperature} \\ \text{Augmentor exit temperature} \\ \text{Main burner fuel flow} \end{bmatrix}$$

TABLE II. COMPARISON OF LQR AND OFR DESIGNS

OFR Feedback Matrix

	x_1	x_2	x_3	x_5	x_{17}
u_1	0.336560	1.42533	0.284484	-0.244595-02	0.196502
u_2	.580896	.191406	-0.332299	-0.528128	-0.526202-01
u_3	35.5966	3.02362	-3.22347	-8.75157	1.28023
u_4	1.53157	-66.1373	12.7661	1.37397	-2.27481
u_5	-0.102381	-0.803223	-1.98148	-0.127121	-0.735746-01

LQR Feedback Matrix

	x_1	x_2	x_3	x_5	x_{17}
u_1	0.497890	2.42002	-0.101360	0.120072	0.240518
u_2	.690374	.211490	.421472-01	-0.553484	-0.765097-01
u_3	36.1351	-6.78935	9.98711	-8.68853	.547046
u_4	1.18785	-23.6908	21.8884	2.04405	.216411
u_5	.837923-01	-0.992363	-0.946234	-0.258270	-0.120667

OFR Cost (J)

The optimum cost = Trace (Ricatti matrix)
55127.5

LQR Cost (J)

The optimum cost = Trace (Ricatti matrix)
55128.8

OFR Closed Loop Eigenvalues

	-554.703	+ J*	15.3219
x_3	-554.703	+ J*	-15.3219
	-52.7604	+ J*	3.14720
	-52.7604	+ J*	-3.14720
	-46.5396	+ J*	.000000
	-40.2390	+ J*	.000000
	-24.6247	+ J*	.000000
	-18.1467	+ J*	3.89611
	-18.1467	+ J*	-3.89611
	-21.3023	+ J*	.000000
	-18.8500	+ J*	.000000
x_2	-9.65625	+ J*	2.31956
x_5	-9.65625	+ J*	-2.31956
	-0.669755	+ J*	.000000
x_1	-6.14437	+ J*	.000000
	-1.97937	+ J*	.000000
x_{17}	-10.5293	+ J*	.000000

LQR Closed Loop Eigenvalues

x_3	-349.868	+ J*	0.000000
x_2	-10.0215	+ J*	2.96296
x_5	-10.0215	+ J*	-2.96296
x_{17}	-10.6641	+ J*	.000000
x_1	-6.13386	+ J*	.000000

TABLE III. COMPARISON OF THREE CONTROL STRUCTURES

Structure #1

	x_1	x_2	x_3	x_5	x_{17}
u_1	0.331103	1.42786	0.241943	0.106119-02	0.196665
u_2	.563311	.183079	-0.278551	-0.531235	-0.538824-01
u_3	35.5320	2.94085	-2.64476	-8.81418	1.27011
u_4	1.38780	-65.6406	11.3587	1.54749	-2.22466
u_5	-0.800610-01	-0.846198	-1.73410	-0.153524	-0.784924-01

Structure #2

	x_1	x_3	x_5	x_{17}
u_1	0.724593	3.35082	-0.574654	-0.129167
u_2	.621300	.672284-01	-0.603105	-0.973783-01
u_3	35.7266	1.91687	-9.97863	.477242
u_4	-13.7957	-124.472	27.1895	13.8395
u_5	-0.196554	-2.82217	.122541	.186423

Structure #3

	x_1	x_2	x_3	x_5	x_{17}	x_{14}
u_1	0.324920	1.41944	0.251627	-0.676767-02	0.184383	0.548166-01
u_2	.582912	.182919	-0.278377	-0.531972	-0.546010-01	.617017-02
u_3	35.4674	2.84734	-2.58472	-8.93478	1.11033	.806580
u_4	1.50482	-65.4595	11.2330	1.80488	-1.90962	-1.65499
u_5	-0.795668-01	-0.845408	-1.73433	-0.152529	-0.770434-01	-0.655999-02

OFR Cost (J)

Structure 1	55 237.8
Structure 2	58 164.8
Structure 3	55 233.8